

# Technical Comments

## Comment on “Simple Equations for Helical Vortex Wakes”

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WOOD<sup>1</sup> has presented a formulation for the efficiency of a propeller or rotor that has an infinite number of blades or, equivalently, is represented by an actuator disc, based, respectively, on simple vortex and momentum theory. In this work, an arbitrary “thermodynamic efficiency” is introduced to account for irreversible conversions of energy by viscosity which are said to be omitted in “most ‘aerodynamic’ analyses.”

Long-established methods of propeller analysis<sup>2–4</sup> give for  $\eta$ , the propeller efficiency of a blade element

$$\eta = \frac{(1 - a')}{(1 + a)} \frac{\tan \phi}{\tan(\phi + \gamma)}$$

where  $a$  is the axial induced velocity factor in the far wake,  $a'$  is the radial induced velocity factor in the far wake such that the induced axial velocity is  $aV$  and the induced tangential velocity is  $a'\Omega r$ . Also,

$$\tan \phi = \frac{V}{\Omega r} \frac{1 + a}{1 - a'}$$

$$\tan \gamma = \frac{C_d}{C_l}$$

where  $V$  is the forward velocity,  $\Omega$  is the angular rotational speed,  $r$  is the radius distance from the axis of rotation,  $C_d$  is the propeller section profile drag coefficient, and  $C_l$  is the propeller section lift coefficient. Thus, it is clear that an efficiency  $\eta_s$ , arising from the action of the propeller blades, represents the major contribution to the undefined irreversible energy losses of Ref. 1, where

$$\eta_s = \frac{\tan \phi}{\tan(\phi + \gamma)}$$

Similar results can be obtained for hovering rotors. Glauert<sup>3</sup> showed that for a representative propeller, the calculated values of integrated efficiency associated with profile drag losses were between 80–90% over most of the operating range. Using calculated values in an example given by Weick,<sup>4</sup> values of  $\eta_s$  between 0.85–0.92 are obtained for blade sections from 0.3 to 0.90 of the radius. It is to be noted that the values of thermodynamic efficiency obtained empirically from test results in Fig. 2 of Ref. 1 lie in these ranges.

The need to account for vortex-induced flow velocity in the wake of propellers and rotors in determining the helical pitch of the vortices as indicated in Fig. 1 of Ref. 1 is also generally recognized. Theodorsen<sup>5</sup> pointed out that this is an essential element in the application of the theory of optimum propeller load distribution to heavily loaded propellers. Also, it has been shown<sup>6</sup> that the vortex and momentum theories for hovering rotors and propellers having infinite numbers of blades lead to identical results only when the effect of vortex-induced velocity on the helical pitch in the far wake is accounted for in the vortex theory.

## References

- <sup>1</sup>Wood, D. H., “Simple Equations for Helical Vortex Wakes,” *Journal of Aircraft*, Vol. 31, No. 4, 1994, pp. 994, 995.
- <sup>2</sup>Glauert, H., *The Elements of Airfoil and Airscrew Theory*, 2nd ed., Cambridge Univ. Press, Cambridge, England, UK, 1947, pp. 208–219.
- <sup>3</sup>Glauert, H., “Airplane Propellers,” *Aerodynamic Theory*, edited by W. F. Durand, Vol. IV, Div. L, Durand Reprinting Committee, California Inst. of Technology, Pasadena, CA, 1948, pp. 191–198, 201–225, 230–239.
- <sup>4</sup>Weick, F. E., *Aircraft Propeller Design*, McGraw-Hill, New York, 1930, pp. 64–81.
- <sup>5</sup>Theodorsen, T., *Theory of Propellers*, McGraw-Hill, New York, 1946, pp. 6–10.
- <sup>6</sup>Flax, A. H., “Vortex and Momentum Theories for Hovering Rotors,” *AIAA Journal*, Vol. 21, No. 11, 1983, pp. 1595, 1596.

## Reply by the Author to A. H. Flax

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CONVENTIONAL analyses of propellers or rotors, leading, e.g., to the equations given by Flax,<sup>1</sup> are based on the one-dimensional conservation equations for mass, momentum, angular momentum, and energy for the airflow through the blade disc. The main difficulty appears to be the exclusion of the possibility that significant amounts of the conserved quantities reside in the trailing vortices, in particular the tip vortices.

The purpose of the Note was to consider Eq. (1), relating  $p_z$ , the pitch of the tip vortices behind a finite number of blades, to the average axial velocity in the far-wake  $U_z$ , within the framework of conventional analyses. To the author's knowledge, Eq. (1) is new, although it is an immediate consequence of Eq. (8) of Hardin.<sup>2</sup> The primary outcome of the Note is Eq. (6), a relation between  $p_z$  and the advance ratio  $J$ , derived from Eq. (1) using the assumption that some of the energy and angular momentum supplied by the blades

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